Rigorous and Simplified Models for the Capacitance of a Circularly Symmetric Via

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***Abstract*— An integral-equation method is presented for analyzing the capacitance of circularly symmetric vias with thick conductors and conical posts. From the numerical data, simple expressions are derived which account for the effect of conductor thickness and cone angle. Use of the approximate expressions allows one to accurately determine via capacitance while using a simpler thin-conductor straight-post via model.**

***Index Terms*—Capacitance, numerical analysis, packaging.**

I. Introduction

Vias connect transmission lines residing on different layers of a multilayered board, and hence, typically consist of a conical post or cylinder that may or may not pass through a hole in a ground plane. Most vias are not straight cylinders as sometimes assumed, but are cone shaped with a typical cone angle of about 30°. The via cylinder or cone often has flanges at both ends to facilitate connections to microstrip traces. The via structure is usually embedded within a multilayered dielectric, and it is not uncommon in an integrated circuit for the conductor thickness to be on the same order as the dielectric thickness. A typical via structure is shown in Fig. 1. Vias are usually modeled as a combination of lumped elements. The effect of vias on a circuit is mostly capacitive, especially if the via crosses a ground (or power) plane. At low frequencies, this capacitance can be neglected, but at high frequencies the via capacitance must be taken into account when designing an integrated circuit or multichip module.

Previous investigations have determined the excess capacitance and inductance of a via connecting two lines above the same ground plane [1] and the excess capacitance through a ground plane in a homogeneous medium [2]. In [3], the capacitance of a via through a thick ground plane is determined, and in [4] the analysis is extended to account for the connecting traces. A finite-difference time-domain (FDTD) method is used to determine a lumped-element via model in [5], [6]. In [7], the via capacitance is determined from

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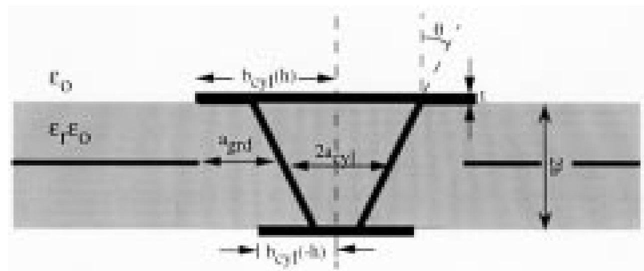

Fig. 1. The cross-sectional view of a conical via through a dielectric slab.

an integral-equation method whose Green's function accounts for the dielectric. In [8], [9], Laplace's equation is solved by separation of variables and domain decomposition to account for a multilayered dielectric.

In this paper, we extend the integral-equation method employed in [7] to account for conductor thickness and the conical shape of vias. In addition, simplified approximate formulas for capacitance are developed from the numerical data which account for conductor thickness and cone angle. Use of these simplified formulas allows one to approximate the conical post with a straight cylinder of predetermined radius, thus enabling one to take advantage of more efficient techniques such as those of [8], [9]. Regardless of the method, the accurate modeling of thick conductors increases the number of unknowns and, therefore, the computation time.

II. Integral Equations

Assuming the via post and flanges are charged to a potential of V and the ground plane is held at zero potential, one readily formulates the coupled integral equation [7]

$$\int_{\mathcal{C}_c} q^c(\ell')G(\rho,z;\rho',z')\,d\ell' + \int_{\mathcal{C}_g} q^g(\ell')\,G(\rho,z;\rho',z')\,d\ell' = V, \quad (\rho,z)\in\mathcal{C}_c$$

$$\int_{\mathcal{C}_c} q^c(\ell')G(\rho,z;\rho',z')\,d\ell' + \int_{\mathcal{C}_g} q^g(\ell')\,G(\rho,z;\rho',z')\,d\ell' = 0, \quad (\rho,z)\in\mathcal{C}_g \quad (1)$$

where ρ' and z' are functions of the arc displacement, ℓ', q^c, and q^g are the charge density on the central post and the ground plane, respectively, and $\mathcal{C}_c$ and $\mathcal{C}_g$ are the contour of the central cylinder and flanges and the contour of the ground plane, respectively. Ideally, the contour of the ground plane extends to infinity, but numerically the contour extends to a radius where the charge density has sufficiently decayed so that it can be truncated with a negligible effect on the potential. A radius of $\rho = 10b_{\rm cyl}$ is usually sufficient. The Green's function (G) is the potential due to a ring of charge in the presence of a dielectric slab, and is given by

$$G(\rho, z; \rho', z') = k(\rho, z; \rho', z') + g(\rho, z; \rho', z') \quad (2)$$

where

$$k(\rho, z; \rho', z) = -\frac{1}{2\pi\epsilon} \frac{\beta\rho'}{\sqrt{\rho\rho'}} K(\beta) \quad (3)$$

and

$$g(\rho, z; \rho', z') = \begin{cases} g_d(\rho, z; \rho', z'), & \text{if } z' \in (-h, h) \\ g_0(\rho, z; \rho', z'), & \text{otherwise.} \end{cases} \quad (4)$$

The permittivity ϵ of (3) is the permittivity of the medium in which the ring of charge resides, κ is the complete elliptic integral of the first kind, and β is

$$\beta = \sqrt{\frac{4\rho\rho'}{(\rho+\rho')^2 + (z-z')^2}}. \quad (5)$$

The function g_d is defined as

$$\begin{aligned} g_d(\rho, z; \rho', z') = &-\frac{(\epsilon_r - 1)}{2\epsilon_r\epsilon_0} \int_0^\infty \rho' J_0(\rho'\kappa) \\ &\cdot \left[\frac{\cosh(\kappa z')e^{-\kappa h}}{\epsilon_r \cosh(\kappa h) + \sinh(\kappa h)} \cosh(\kappa z) \right. \\ &\left. + \frac{\sinh(\kappa z')e^{-\kappa h}}{\epsilon_r \sinh(\kappa h) + \cosh(\kappa h)} \sinh(\kappa z) \right] \\ &\cdot J_0(\rho\kappa)\, d\kappa, \qquad z \in (-h, h) \end{aligned} \quad (6)$$

$$\begin{aligned} g_d(\rho, z; \rho', z') = &-\frac{(\epsilon_r - 1)}{2\epsilon_r\epsilon_0} \int_0^\infty \rho' J_0(\rho'\kappa) \\ &\cdot \left[\frac{\cosh(\kappa z')\cosh(\kappa h)e^{-\kappa|z|}}{\epsilon_r \sinh(\kappa h) + \cosh(\kappa h)} \right. \\ &\left. + \frac{\text{sign}(z)\sinh(\kappa z')\sinh(\kappa h)e^{-\kappa|z|}}{\epsilon_r \cosh(\kappa h) + \sinh(\kappa h)} \right] \\ &\cdot J_0(\rho\kappa)\, d\kappa, \qquad |z| > h. \end{aligned} \quad (7)$$

The homogeneous part of the Green's function outside of the slab is given by

$$\begin{aligned} g_0(\rho, z; \rho', z') = &-\frac{(1 - \epsilon_r)}{2\epsilon_r\epsilon_0} \int_0^\infty \rho' J_0(\rho'\kappa) \\ &\cdot \left[\frac{\tanh(\kappa h)\cosh(\kappa z)e^{-\kappa|z'|}}{1 + \epsilon_r \tanh(\kappa h)} \right. \\ &\left. + \frac{\text{sign}(z')\sinh(\kappa z)e^{-\kappa|z'|}}{\epsilon_r + \tanh(\kappa h)} \right] \\ &\cdot J_0(\rho\kappa)\, d\kappa, \qquad z \in (-h, h) \end{aligned} \quad (8)$$

$$\begin{aligned} g_0(\rho, z; \rho', z') = &-\frac{(1 - \epsilon_r)}{2\epsilon_r\epsilon_0} \int_0^\infty \rho' J_0(\rho'\kappa) \\ &\cdot \left[\frac{\sinh(\kappa h)e^{-\kappa(|z|+|z'|-2h)}}{1 + \epsilon_r \tanh(\kappa h)} \right. \\ &\left. + \frac{\text{sign}(zz')\sinh(\kappa h)e^{-\kappa(|z|+|z'|-2h)}}{\epsilon_r + \tanh(\kappa h)} \right] \\ &\cdot J_0(\rho\kappa)\, d\kappa, \qquad |z| > h. \end{aligned} \quad (9)$$

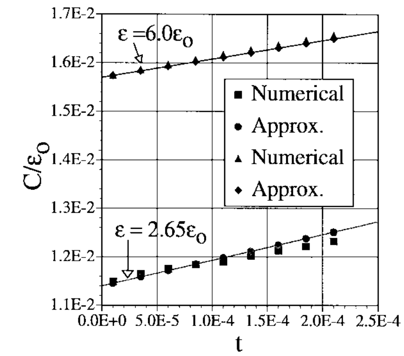

Fig. 2. The capacitance of a cylindrical via without a ground plane with respect to lip thickness, t. The dimensions are $a_{\rm cyl} = 0.4$ mm, $b_{\rm cyl} = 0.6$ mm, and $2h = 0.8$ mm.

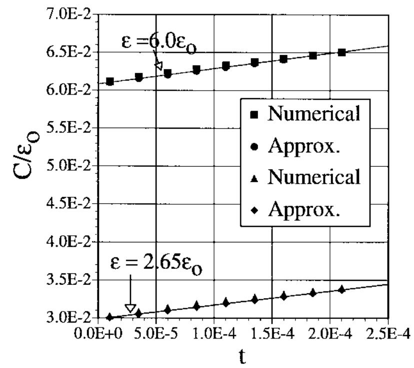

Fig. 3. The capacitance of a cylindrical via through a ground plane with respect to lip thickness, t. The dimensions are $a_{\rm cyl} = 0.4$ mm, $a_{\rm grd} = 0.6$ mm, $b_{\rm cyl} = 0.6$ mm, and $2h = 0.8$ mm.

A pulse expansion and point-matching scheme is used to convert the coupled integral equations to a matrix equation [7], [10]. In [7], the singularity in the Green's function when the distance between the source and observation point tends toward zero is handled by extracting the singularity and analytically integrating it. The singularity subtraction method derived in [7] is only valid as the observation point approaches the source point along the lines $z = z'$ or $\rho = \rho'$. In the present analysis, integration over the logarithmic singularity in the elliptic integral employs the *linlog* quadrature rule developed in [11]. Not only is this approach simpler to implement than the singularity subtraction method, it allows one to integrate over the singularity along any contour, including the cone under consideration. In addition, the homogeneous part of the Green's function has a simple dependence on ρ' and z'. If the contour of integration is completely in the ρ- or z-direction, then considerable improvement in efficiency is obtained by integrating the homogeneous part of the Green's function (g) analytically.

III. Results

The capacitance of a straight cylindrical via not passing through a ground plane versus flange thickness is shown in Fig. 2 for two different relative permittivities, $\epsilon_r = 2.65$ and $\epsilon_r = 6.0$. In Fig. 3, the capacitance of a straight cylindrical via passing through a ground plane is shown for the same

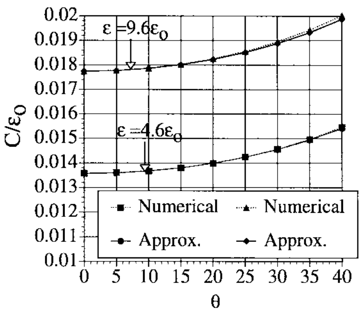

Fig. 4. The capacitance of a conical via without a ground plane with respect to cone angle θ. The dimensions are $a_{\rm cyl} = 0.6$ mm, $b_{\rm cyl} = 0.2$ mm $+ a_{\rm cyl}$, and $2h = 1.0$ mm.

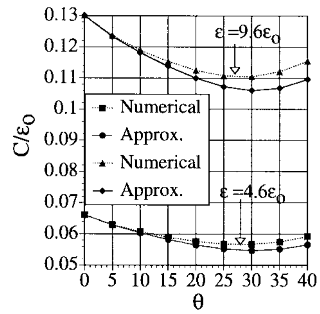

Fig. 5. The capacitance of a conical via through a ground plane with respect to cone angle, θ. The dimensions are $a_{\rm cyl} = 0.6$ mm, $b_{\rm cyl} = 0.2$ mm $+ a_{\rm cyl}$, $a_{\rm grd} = b_{\rm cyl}$, and $2h = 1.0$ mm.

permittivities. A straight cylindrical via has an angle of incline of $\theta = 0°$, therefore, we define $b_{\rm cyl} = b_{\rm cyl}(h) = b_{\rm cyl}(-h)$. For all cases shown, the flange radius is $a_{\rm cyl} = 0.4$ mm, $b_{\rm cyl} = 0.6$ mm, and the dielectric thickness is $2h = 0.8$ mm. For the geometries with a ground plane, the opening has a radius of $a_{\rm grd} = b_{\rm cyl}$, and for the geometries without a ground plane, the ground reference is at $\rho = \infty$. The data from the integral equation model is shown in the curves labeled *Numerical*. In all the cases, the capacitance has a nearly linear variation with flange thickness. Since the variation is linear, an approximate formula can be obtained to relate via capacitance and lip thickness.

The capacitance can be approximated by the formula $C = Mt + C_{\rm th}$, where t is the thickness of the connecting flanges and $C_{\rm th}$ is the capacitance of the via with vanishingly thin flanges. The value of the slope M is empirically obtained. If the via does not pass through a ground plane, M is approximately

$$M = (0.0120\epsilon_r^2 - 0.236\epsilon_r + 1.04)\left(\frac{b_{\rm cyl}}{h}\right) + 0.0170\epsilon_r^2 - 0.392\epsilon_r + 5.434. \quad (10)$$

For a via passing through a ground plane, the value of M is given by

$$M = 6.55\left[\epsilon_r^{.161}\left(\frac{b_{\rm cyl}}{h}\right) + \epsilon_r^{.0511}\right]. \quad (11)$$

In Figs. 2 and 3, the value of capacitance found from $C = Mt + C_{\rm th}$ is shown compared with those computed, which are the curves labeled *Approx.*

In Fig. 4, the capacitance of a conical via without a ground plane with respect to cone angle is shown for two different permittivities $\epsilon_r = 4.6$ and $\epsilon_r = 9.6$. The ground reference for this case is located at infinity. The center radius of the cylinder is held at $a_{\rm cyl}(0) = 0.6$ mm, and the connecting pads add an additional 0.2 mm to the lip radius $b_{\rm cyl}$. The thickness of the dielectric is $2h = 1.0$ mm. The curves labeled *Numerical* show the capacitance as determined by the integral model. A common approximation of the conical cylinder is a straight cylinder having the average radius $\tilde{a}_{\rm cyl} = \frac{1}{2}[a_{\rm cyl}(h) + a_{\rm cyl}(-h)]$. This approximation is equivalent to $\theta = 0°$; thus, one can see the error of this approximation. A better approximation is

$$\tilde{a}_{\rm cyl} = [a_{\rm cyl}^{2.5}(h) + a_{\rm cyl}^{2.5}(-h)]^{1/2.5} \quad (12)$$

and the capacitance of a straight via using the approximate cylinder radius is shown by the curve labeled *Approx.* in Fig. 4. The maximum error for the cases shown is 0.25%.

The capacitance of a conical via passing through a ground plane is presented in Fig. 5 for the relative permittivities $\epsilon_r = 4.6$ and $\epsilon_r = 9.6$ for comparison with the capacitance shown in Fig. 4. The center radius of the cylinder is again held at $a_{\rm cyl}(0) = 0.6$ mm, and the lip radius is $b_{\rm cyl}(\pm h) = a_{\rm cyl}(\pm h) + 0.2$ mm. The ground plane is in the $z = 0$ plane and has an opening of radius $a_{\rm grd} = b_{\rm cyl}(h)$. Again the curves labeled *Numerical* show the results from the exact model. The presence of a ground plane complicates the problem since either the flanges-to-ground capacitance or the cylinder-to-ground capacitance could dominate. If the cylinder–ground capacitance dominates, the radius approximation

$$\tilde{a}_{\rm cyl} = a_{\rm grd} - \min(a_{\rm grd} - a_{\rm cyl}) \quad (13)$$

works reasonably well. The via capacitance based on this approximation is shown in the curves labeled *Approx.* in Fig. 5. One notes that as the cone angle decreases, the approximation breaks down, which is due to the increasing importance of the flange-to-ground capacitance.

IV. Conclusions

A method for determining the capacitance of conical vias with thick conductors is presented. Since the via capacitance displays a linear dependence on conductor thickness, a simple empirical expression for capacitance can be found and is presented. Use of the simple approximate expression allows one to use a simpler thin conductor model which requires less computational effort. For a via not passing through a ground plane, the central processing unit (CPU) time was reduced from 13.4 to 3.85 s by using the approximation in conjunction with the thin conductor model. For conical vias, approximating the conical central cylinder with a straight cylinder makes the problem easier to model and works well, particularly if the via does not pass through a ground plane.

References

[1] T. Wang, J. R. Mautz, R. F. Harrington, "Quasi-static analysis of a microstrip via through a hole in a ground plane," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 1008–1013, June 1988.

[2] T. Wang, J. R. Mautz, and R. F. Harrington, "The excess capacitance of a microstrip via in a dielectric substrate," *IEEE Trans. Computer-Aided Design*, vol. 9, pp. 48–56, Jan. 1990.

[3] P. Kok and D. DeZutter, "Capacitance of circular symmetric model of a via hole including finite ground plane thickness," *IEEE Trans. Microwave Theory Tech.,* vol. 39, pp. 1229–1234, July 1991.
[4] P. Kok and D. DeZutter, "Prediction of the excess capacitance of a via-hole through a multilayered board including the effect of connecting microstrips and striplines," *IEEE Trans. Microwave Theory Tech.,* vol. 42, pp. 2270–2276, Dec. 1994.
[5] W. Becker, P. Harms, and R. Mittra, "Time-domain electromagnetic analysis of interconnects in a computer chip package," *IEEE Trans. Microwave Theory Tech.,* vol. 40, pp. 2155–2163, Dec. 1992.
[6] P. Harms, J. Lee, and R. Mittra, "Characterizing the cylindrical via discontinuity," *IEEE Trans. Microwave Theory Tech.,* vol. 41, pp. 153–156, Jan. 1993.
[7] A. W. Mathis and C. M. Butler, "Capacitance of a circularly symmetric via for a microstrip transmission line," *Proc. Inst. Elect. Eng., Microwave, Opt., Antennas,* vol. 141, pt. H, pp. 265–271, Aug. 1994.
[8] F. Tefiku and E. Yamashita, "Efficient method for the capacitance calculation of circularly symmetric via in multilayered media," *IEEE Microwave Guided Wave Lett.,* vol. 5, pp. 305–307, Sept. 1995.
[9] F. Tefiku and E. Yamashita, "Capacitance characterization method for thick-conductor multiple planar ring structures on multiple substrate layers," *IEEE Trans. Microwave Theory Tech.,* vol. 40, pp. 1894–1902, Oct. 1992.
[10] R. Harrington, *Field Computation by Moment Methods.* New York: Macmillan, 1968.
[11] J. Ma, V. Rokhlin, and S. Wandzura, "Generalized Gaussian quadrature rules for systems of arbitrary functions," *SIAM J. Numer. Anal.,* vol. 33, no. 3, pp. 971–996, June, 1996.

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